Knowledge Representation and ontologies from the logical point of view

Marie Duží
VŠB-TU
Ostrava

Pavel Materna
FÚ AV ČR
Praha

Content

- Introduction: basic notions and problems
- Knowledge representation languages
  - First Order Predicate Logic (FOL)
    - Limitations, paradoxes
  - Epistemic, modal, intensional logics
- Transparent Intensional Logic
- Ontology languages and Semantic web
- Conclusion

Problems, basic notions

- Ontologies – what is it?
  - formalization, conceptualization, hierarchy of entities, concepts, knowledge, conceptual analysis, ... ??
- Concept – what is it?
  - conceptual analysis
- Knowledge – what is it?
  - How can knowledge be represented and hierarchically ordered?
  - Problem: knowledge representation
- Languages apt for
  - Knowledge representation and semantic web
- Logical analysis of natural language
  - Inferential machine

Problems, basic notions

- Ontology in philosophy: science of being

  (*what* is there, in the world, *what* can we talk about)
- Ontologies in informatics (broader sense):
  - Formal description of particular domains:
    - *What* can we talk about here
    - *How* can we describe it
  - Hierarchical description of that, classification

Building ontologies

- *Horrocks*: hierarchical description of the important ***concepts*** of the domain together with the description of the properties of their instances
- Conceptualization, conceptual analysis:
  What is there ?
  How can we describe it ?
  What can we *deduce* from that ?

Knowledge representation languages

- Formalization should be *meaning driven*
- Should make it possible to analyse particular statements in such a way that the *inference machine* could derive *all the logical consequences* (of a knowledge base)
- Hence the inference machine shouldn't:
  - "over-infer"
    (derive something that does not follow)
  - "under-infer"
    (not to be able to derive something that does follow)

Language of first-order predicate logic

FOL limitations:

- Model theoretic approach:
  - Appropriate for mathematics (“stenography of math.”)
- Universe of discourse – a set of individuals
- We can talk about individuals,

  expressing their properties and relations between individuals
- We cannot talk about properties of (individuals, properties, functions, relations, properties of ***concepts,*** …), unless they are members of the (*many-**sorted***) universe

Language of the first-order predicate logic (FOL)

- Expressive power of the language does not enable us to analyze expressions in a fine-grained enough way, which leads to $\rightarrow$
- Paradoxes of analyses:

From *true* assumptions

using only correct (i.e. *truth-preserving*) rules of deduction,

we obtain an obviously *false*, or even non-reasonable, conclusion.

Coarse-grained analysis paradoxes

- *It is necessary that 9 > 7.*
  The number of planets is 9.

  hence

  It is necessary that the number of planets is > 7 ??
- Out of the scope of FOL
- Modal logic: $\Box(9 > 7)$
  - $n(pl) = 9$
  - *hence* $\Box(n(pl) > 7)$

What is the semantics of the box?

Is the conclusion *necessary*?

It could be otherwise, it is a contingent (*non-necessary*) fact !

Coarse-grained analysis paradoxes

- *The president of CR is an economist*
- FOL: E(p(cr)) – either true or false
- But in February 2003 ***there is no president*** *of CR*.
  If the sentence is false, then it is true that the president of CR is not an economist, which means that ***there is a president*** – paradox.
- Logic of *partial functions* is needed

Coarse-grained analysis paradoxes

- *John Kerry wants to become the President of USA*
  The President of USA is George W. Bush
  hence
  John Kerry wants to become George W. Bush !?!
- FOL: W(j, p(u))

  $$\frac{W(j, p(u)) \qquad g = p(u)}{W(j, g)}$$

- p(u) denotes an individual, the second premise states an identity of individuals; however, John Kerry does not want to become an individual, he wants to hold the *office* of the president
- Higher-order (intensional) logic (HOL) is needed

Coarse-grained analysis paradoxes

- *Oidipus seeks the murderer of his father*
  Oidipus is the murderer of his father
  hence
  Oidipus seeks himself
- Oidipus does not seek himself (he is not a schizophrenic), he wants to know who plays the *role* of the murderer
- Higher-order (intensional) logic (HOL) is needed

Coarse-grained analysis paradoxes

- *Charles calculates 2+5*
  2+5 = 7
  hence
  Charles calculates 7 ??
- Charles is related to the *procedure* of adding the two numbers, not to the result of the procedure
- Hyper-intensional logic with procedural semantics is needed

Coarse-grained analysis paradoxes

- *Charles believes that Prague has 1.048.576 inhabitants*

  $1.048.576 = 100.000_{(16)}$

Yet, *Charles does not have to believe that Prague has $100.000_{(16)}$ inhabitants*

- Not solvable by epistemic intensional logics (relating Charles to the proposition that …)
- By his knowing Charles is related to the *mode of presentation* (***construction***) of the proposition
- Hyper-intensional logic with procedural semantics is needed

Coarse-grained analysis

1) *Charles knows **that** the President of CR is the commander in chief of the Czech army*
2) *Charles knows **of** the President of CR that he is the commander in chief of the Czech army*

- The two sentences are logically independent !
  1) is not entailed by 2),
  2) is not entailed by 1).
- Higher-order intensional logic is needed

Extensions of FOL

- Modal, intensional, dynamic, epistemic, … logics
- These logics provide theories: *ad hoc* axioms and rules that define a set of models;
  - “syntactically driven axiomatization”
    they do not provide a hint on which particular problem they should be applied
- Ontology language should be universal, highly expressive, with *transparent semantics* and *meaning driven axiomatisation* !

Extensions of FOL: formalising reasoning intelligent agents

- A rational agent in a multi-agent world is:
  - able to reason about the world (what holds true), and
  - about its own cognitive state, and
  - about that of other agents
- **The theory has to be able to**
  - ‘talk about’ and quantify over the objects of propositional attitudes
  - iterate attitudes of distinct agents (*a* knows that *b* knows that …)
  - express self-referential statements (*a* knows that *a* knows, …)
  - respect limitations of particular agents

An agent a is resource-bounded

- Lack of inferential capacity
- Lack of computational resources
  - Refusal to acknowledge valid consequences
  - Lack of motivation
  - Bounded material resources like
    - time for calculation and derivation
    - storage capacity
  - and so on ...
- *Out of the scope of FOL*

Epistemic logics: K(now), B(elieve)

- All of them:
  - Predicate logic axioms and rules +
  - (**K**) **$K\varphi \wedge K(\varphi \Rightarrow \psi) \Rightarrow K\psi$ (logical rationality)**
  - (**N**) From a formula **φ derive $K\varphi$**. (Necessitation)
- Some of them:
  - (T) $K\varphi \Rightarrow \varphi$ knowledge implies truth
  - (D) $K\varphi \Rightarrow \neg K\neg\varphi$ knowledge is consistent
  - (4) $K\varphi \Rightarrow KK\varphi$ positive introspection
  - (5) $\neg K\varphi \Rightarrow K\neg K\varphi$ negative introspection
  - **Belief usually defined: $\neg K\varphi =_{df} B\neg\varphi$**

Epistemic Closure problem

- *a* knows that φ;
  a knows that φ implies ψ

  hence *a* knows that φ implies ψ:

  $[K_a \varphi \wedge K_a(\varphi \supset \psi)] \supset K_a \psi$
- An agent is related to a set of ***propositions***
- ***Paradox of omniscience***:
- *a* knows that Bill is walking →
  a knows that Bill is walking and
  arithmetic is not recursively axiomatizable and
  whale is a mammal and …
- Agent *a* is either an analytic genius or *a* does not know what *a* knows

Applicability

- *Kripke* intensional semantics

too strong version of the Closure:

- if $K_a\varphi$ and $\varphi \models \psi$, then $K_a\psi$.
  - The agent knows all the logical consequences of his explicit knowledge
  - and all the analytically true propositions
- *a* knows, e.g., that

arithmetic is not completely axiomatizable,
all whales are mammals, …

Applicability

- *Montague-Scott* domains (intensional semantics):
  - if K_a φ and φ ⇔ ψ, then K_a ψ.
- More realistic version: *a* knows all the ***equivalent***s of his explicit knowledge
- But: if *a* knows that Bill walks then *a* knows that Bill walks and arithmetic is not complete, and …
- Instances of the Paradox of (logical / mathematical) *omniscience*

Three kinds of knowing

- Epistemology—two kinds of knowledge
  - ***explicit knowledge***: the agent *a* is aware of; built in his memory / knowledge base
  - ***implicit knowledge***: ascribed to the agent *a* from outside; a set of propositions that *a* cannot falsify
- But we need primarily
  - ***inferable knowledge***: that the agent *a* is ***able*** to infer from the explicit knowledge base

Intensional semantics

(Modal) epistemic logics with
Kripke, Montague-Scott, etc., semantics

inevitably lead to the paradox of omniscience;

the tightest restriction -- logical equivalence

Apt for modelling ***implicit knowledge*** of a ***passive*** agent

Not-usable *when modelling a multi-agent system of* ***resource-bounded*** *intelligent autonomous agents which* ***act,*** *but are* ***not omniscient***

What remains; syntactic approach ?

- Technical objections:
  - It is prone to **inconsistency**
    (stemming from ***self-referential*** statements and the necessity to ***mention formulas*** *within* the theory):
    - Tarski's biconditional: True 'φ' ⇔ φ; for all φ
    - diagonalisation lemma: there is a φ, ¬True 'φ' ⇔ φ
    - ⇒ Liar paradox
  - Avoided by:
    - hierarchy of (meta-)languages -- 'grounded' Truths, or
    - hierarchy of 'stable' truth predicates
- ***Major philosophical objection:*** an agent is not related to a formula, term, but to the ***meaning*** of the embedded clause

Higher-order framework

- We need a highly expressive language, with transparent natural semantics
- Appropriate for natural language analysis
- We have such a framework at our disposal:

Transparent Intensional Logic

Transparent Intensional Logic

- *Procedural semantics* and theory of concepts
- Concepts are *structured meanings*
  - known as TIL ***constructions***
  - Procedures ***structured*** from the ***algorithmic*** point of view
- Specification language appropriate for knowledge representation
- Inferable/explicit knowledge concerns *meanings*,

  and only derivatively the outputs of the procedures (if any): e.g., possible-world propositions
  - The sense of the embedded clause: specify the *way* of evaluating truth-conditions in ***any w,t***

Procedural theory of meanings

- Obviously, any *set-theoretical* theory of concepts (e.g. Fregean) cannot be competent to handle *structured meanings*:
  - ‘there is nothing about a set in virtue of which it may be said to present something’ (Zalta);
  - each (general) concept is in such a theory identified with the respective set.
  - We wish to distinguish between a *concept* of an entity and the entity itself
- Hence:
  meaning = algorithmically structured procedure = TIL construction

Procedural theory of meanings

- Pavel Tichý (1968): 'Sense and Procedure', later in 'Intensions in terms of Turing machines' formulated the idea of ***structured meanings***: meaning of an expression is a procedure (structured from the ***algorithmic*** point of view), a way of arriving at the denoted entity; ***TIL construction***
- Pavel Tichý (1988): *The Foundations of Frege's Logic*
- Pavel Materna (1998): 'Concepts and Objects': ***concept*** is a closed construction
- Y. Moschovakis (1994, 2003): sense and denotation as ***algorithm*** and *value*

Ontology of TIL

- Rich ontology: entities organized in a two-dimensional **infinite ramified hierarchy of** *types*
  - *any entity of any type of any order (even a construction) can be* ***mentioned*** *within the theory* ***without generating paradox***.

Ramified hierarchy of types

- ***1st order:*** algorithmically non-structured entities
  - Basic types: o = {True, False}
    ι = individuals (universal universe of discourse)
    τ = time points (real numbers)
    ω = possible worlds (consistent maximum sets of facts)
  - Functional types: sets of partial *functions* (mappings): $(\alpha_1,\ldots,\alpha_n) \rightarrow \beta$ denoted $(\beta\ \alpha_1\ldots\alpha_n)$.
  - Rule increasing mereological complexity ("horizontal"):
    - If $\alpha_1,\ldots,\alpha_n$, β are types of order *n*, then $(\beta\ \alpha_1\ldots\alpha_n)$ is a type of order *n*.

(α-)**set**s are mapped by characteristic functions – $(o\alpha)$.

Remark

- TIL is an open-ended system.
  The above *epistemic base* {ο,ι,τ,ω} was chosen, because it is apt for natural-language analysis;
  but in the case of mathematics, a (partially) distinct base would be appropriate; for instance, the base consisting of natural numbers, of type ν, and truth-values of type ο: {ν, ο}

Ramified hierarchy of types

- ***2nd order***: (algorithmically) structured entities — *Constructions* of 1st order entities, all of them belong to atomic type $*_1$:
  - **Variables:** *x, y, z ...* → any type (not only individuals!)
  - **Trivialisation:** ^{0}**X** → object X
  - **Closure:** $[\lambda x_1 \ldots x_n\ C]$ → Function / $(\beta\ \alpha_1 \ldots \alpha_n)$

    $\alpha_1 \quad \alpha_n \quad \beta$
  - **Composition:** $[C \quad X_1 \ldots X_n]$ → Value of the function

    $(\beta\ \alpha_1 \ldots \alpha_n) \quad \alpha_1 \quad \alpha_n \quad \beta$

\+ molecular types (horizontal rule): sets of functions involving constructions: $(\alpha_1\ \alpha_2 \ldots \alpha_n)$, $\alpha_i = *_1$

Ramified hierarchy of types

3rd order:

- *Atomic type* $*_2$ — ***constructions*** *of 1st or 2nd order entities* (all of them belong to type $*_2$) and
- "horizontal rule" of creating *Molecular types* $(\alpha_1\ \alpha_2\ldots\alpha_n)$, $\alpha_i = *_2$

And so on: $*_3$, $*_4$, ..., $*_n$, ..., $(o\ *_n)$, $(\alpha\ *_m\ *_n)$, …

Examples:

- The set of closed constructions of order 1: Closed / $(o\ *_1)$
- Calculating / $(o\ \iota\ *_n)_{\tau\omega}$

Examples

- The function **+**, defined on natural numbers (of type ν), is not a construction. It is a mapping of type $(\nu\ \nu\nu)$, i.e., a set of triples, the first two members of which are natural numbers, while the third member is their sum.
- The simplest construction of this mapping is ^{0}**+**.
- Closure λ***x*** **[**^{0}**+** ***x*** ^{0}**1]** constructs the successor function.
- The composition of this closure with ^{0}5, i.e., **[**λ***x*** **[**^{0}**+** ***x*** ^{0}**1]** ^{0}**5]**, constructs the number 6.

Examples

- The composition **[^{0}: *x* ^{0}0]** does not *v*-construct anything for any valuation of *x*; it is *improper*.
- The closure λ***x*** **[^{0}: *x* ^{0}0]** is not improper, as it constructs something, even though it is only a degenerated function (one undefined at all its arguments).

Examples of higher-order constructions

- Members of $*_1$:

  ^{0}+, [^{0}+ *x* ^{0}1], λ*x* [^{0}+ *x* ^{0}1], [λ*x* [^{0}+ *x* ^{0}1] ^{0}5],

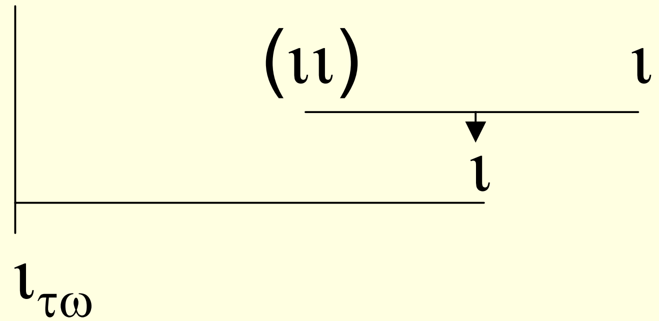
  [^{0}: *x* ^{0}0], λ*x* [^{0}: *x* ^{0}0]
- When IMPROPER / $(o*_1)$ is a set of improper constructions of order 1, the composition **[^{0}IMPROPER ^{0}[^{0}: *x* ^{0}0]** is a member of type $*_2$, and it constructs the truth-value True.
- The constituent **^{0}[^{0}: *x* ^{0}0]** / $*_2$ of this composition is an *atomic proper* construction that constructs **[^{0}: *x* ^{0}0]**, a member of $*_1$.
- It is atomic, because the construction **[^{0}: *x* ^{0}0]** is *not used* here as a constituent *but only mentioned* as an input object.

Intensions vs. extensions

- α-***intension***: member of a type $(\alpha\omega)$
  - frequently $((\alpha\tau)\omega)$, denoted $\alpha_{\tau\omega}$
- α-***extension***: not a function from ω
- Examples of intensions:
  - *student* / $(o\iota)_{\tau\omega}$ — **property** of individuals
  - *the president of CR* / $\iota_{\tau\omega}$ — **individual office**
  - *Charles is a student* / $o_{\tau\omega}$ — **proposition**
  - *age of* / $(\tau\iota)_{\tau\omega}$ — **attribute** (empirical function)
  - *Calculate* / $(o\iota*_1)_{\tau\omega}$ — ***relation-in-intension***

Examples of Intensions

- *Notation*:
  - variables $w \rightarrow \omega$, $t \rightarrow \tau$
  - *composition* $[[C\ w]\ t]$ — C_{wt}
- President of CR / $\iota_{\tau\omega}$ — an individual office constructed by:
- $\lambda w \lambda t\ [{}^0President_{wt}\ {}^0CR]$



Abstr. over *w*,*t*:

TIL: *Shifting Frege-Church scheme*

'The president of CR' → **$\lambda w \lambda t$ [^{0}President$_{wt}$ ^{0}CR]** → **office / $\iota_{\tau\omega}$**

conceptual level (how)

ontological level (what)

(Empirical) expression → **meaning = construction** → **intension (= denotation)**

Nobody in Feb.2003
Vaclav Klaus (now)

"Reference" **Value** of the intension (in w,t)

Out of the scope of logic: result of empirical information retrieval

Method of analysing

consists of the following three steps:

1. *Type-theoretical analysis*: Assign types to the objects talked about, i.e. *only* to those that are denoted by sub-expressions of E; besides, try not to omit any semantically self-contained sub-expression of E (to use *all* of them).
2. *Synthesis*: Compose constructions of these objects so as to construct the object D denoted by E.
3. *Type checking*: Use the assigned types for control so as to check whether the various types are compatible and, furthermore, produce the right type of object in the intended manner.

Examples

- ***The highest mountain is in Asia***
  Mountain/ $(o\iota)_{\tau\omega}$, BAsia/ $(o\iota)_{\tau\omega}$, Highest/ $(\iota\ (o\iota))_{\tau\omega}$, HMA / $o_{\tau\omega}$
- A possible “analysis” is also a trivialisation of the denoted proposition: ^{0}HMA (no good, of course)
- A more fine-grained analysis (but not *the* best)—combining constructions of Mountain, BAsia, Highest as follows:
- $\lambda w\, \lambda t\ [\ [[\ {}^0\text{BAsia}\ w]t]\ \ [\ [[{}^0\text{Highest}\ w]t]\ \ [[{}^0\text{Mountain}\ w]t]\ ]\ ]$

  $(o\ \iota)$ $\quad$ $(\iota\ (o\ \iota))$ $\quad$ $(o\ \iota)$

  ι

  o

  abstraction over *t* : $(o\tau)$
  abstraction over *w* :$((o\tau)\omega)$, abbreviated $o_{\tau\omega}$ (proposition)
- Abbreviated: $\lambda w\, \lambda t\ [\ {}^0\text{BAsia}_{wt}\ [\ {}^0\text{Highest}_{wt}\ {}^0\text{Mountain}_{wt}\ ]]$

Examples

- *Black swans are rare.*

  BlackSwan/$(o\iota)_{\tau\omega}$ Rare/$(o\ (o\iota)_{\tau\omega})_{\tau\omega}$
  Black/$((o\iota)_{\tau\omega}(o\iota)_{\tau\omega})$ Swan/ $(o\iota)_{\tau\omega}$

  - does not, of course, talk about particular individuals, but about *properties*:
- $\lambda w \lambda t\ [{}^0\text{Rare}_{wt}\ {}^0\text{BlackSwan}]$
  - *not fine-grained enough*: $\rightarrow$ the property BlackSwan has to be composed using (adjective modifier) Black and (the property) Swan:
- $\lambda w\ \lambda t\ [{}^0\text{Rare}_{wt}\ \underbrace{[{}^0\text{Black}\ {}^0\text{Swan}]}_{(o\iota)_{\tau\omega}}\ ]$

Solving paradoxes stemming form a coarse-grained analysis of premises

- *Charles calculates 2+5*
  2+5=7
  (FOL)*hence*: *Charles calculates 7* ???
- TIL: Calculating / $(o\iota{*}_1)_{\tau\omega}$ relates Charles not to the number denoted by the term 2+5, but to the *construction* 2+5
  - $\lambda w \lambda t\ [{}^0Calculate_{wt}\ {}^0Charles\ {}^0[{}^0+\ {}^02\ {}^05]]$
  - ${}^0[{}^0+\ {}^02\ {}^05] \neq {}^07$
    distinct constructions, undesirable substitution is blocked

Solving paradoxes (existence)

- *The king of France does not exist*
- FOL: $\neg\exists x\,(x = \mathrm{K(f)})$;

  but K(f) does denote an individual, hence

  $\exists x\,(x = \mathrm{K(f)})$. Contradiction
- TIL: Logic of ***partial functions;*** moreover, 'the King of France' does not denote an individual, but an *individual office* / $\iota_{\tau\omega}$ that is not occupied (does not exist) in *w,t:*

  $\lambda w\, \lambda t\, [\neg\exists x\, [x = \lambda w \lambda t\, [{}^0King_{wt}\ {}^0France]_{wt}]] \Leftrightarrow$

  $\lambda w\, \lambda t\, [\neg[{}^0E_{wt}\ \lambda w \lambda t\, [{}^0King_{wt}\ {}^0France]]]$

Solving paradoxes (partiality)

- *The president of CR is an economist*
- FOL: E(p(cr)) – either true or false
- But in February 2003
  there is no president *of CR*.
  If the sentence is false, then it is true that the president of CR is not an economist, which means that ***there is a president*** – paradox.
- TIL: $\lambda w \lambda t\ [{}^0Economist\ \lambda w \lambda t\ [{}^0Pres_{wt}\ {}^0CR]_{wt}]$
  $\rightarrow$ a proposition / $o_{\tau\omega}$, ***partial function***, undefined (neither true nor false) in some *w,t*

Inferences, paradoxes (modalities)

- *It is necessary that 9 > 7.*
  The number of planets is 9.

hence

It is necessary that the number of planets is > 7

Modal logic: $\Box(9 > 7)$

- $n(pl) = 9$
- *hence* $\Box(n(pl) > 7)$

Solving paradoxes (modalities)

- TIL: (roughly) three kinds of necessity of P
  - Analytical: $\forall w\ \forall t\ P$
  - Nomical: $\lambda w\ \forall t\ P$ (physical laws, etc.)
  - Epistemic: $\forall w\ \forall t\ (Q \Rightarrow P)$ – a logical consequence of known facts Q
- $\forall w\ \forall t\ [{}^0 9 > {}^0 7]$

  $\lambda w\ \lambda t\ [[{}^0\mathrm{Card}\ \lambda x\ [{}^0\mathrm{Planet}_{wt}\ x]] = {}^0 9]$

  contingent fact ($\lambda w\ \lambda t\ \ldots$), no necessity ($\forall w\ \forall t \ldots$)

  Substitution is blocked

Solving paradoxes (attitudes)

- *John Kerry wants to become the President of USA*
  The President of USA is George W. Bush
  hence
  John Kerry wants to become George W. Bush !?!
- TIL: John Kerry is not related to an individual, but to the *office* of the president of USA

  $\lambda w \lambda t\ [{}^0WantBecome_{wt}\ {}^0JohnKerry\ \lambda w \lambda t\ [{}^0Pres_{wt}\ {}^0USA]]$

  $\lambda w \lambda t\ [\ \lambda w \lambda t\ [{}^0Pres_{wt}\ {}^0USA]_{wt} = {}^0GeorgeWBush]$

  Substitution is blocked

Solving paradoxes (attitudes)

- *Charles believes that Prague has 1.048.576 inhabitants*
- *1.048.576 = 100.000*$_{(16)}$
- *Yet, Charles does not have to believe that Prague has 100.000*$_{(16)}$ *inhabitants*
- TIL: Distinct *modes of presentations (constructions)* of one and the same proposition
- By his knowing Charles is related to the ***construction*** of the proposition:
- $^0[\lambda w\ \lambda t\ [[^0\text{Card}\ \lambda x\ [^0\text{Inhabit}_{wt}\ x\ ^0\text{Prague}]] = {}^0 1048576]\ ] \neq$ $^0[\lambda w\ \lambda t\ [[^0\text{Card}\ \lambda x\ [^0\text{Inhabit}_{wt}\ x\ ^0\text{Prague}]] = [^0 16^5]]\ ]$

TIL approach to knowledge representation

Technically as fine-grained as the syntactic approach; two major distinctions:

- an agent is not related to a formula, but to the *meaning* of the embedded clause:

$\lambda w \lambda t\, [{}^0Know_{wt}\ {}^0A\ {}^0[\lambda w \lambda t\, [{}^0Card\ \lambda x\, [{}^0Inh_{wt}\ x\ {}^0Prague] = {}^0 1048546]]]$

$*_1$

Know / $(o\ \iota\ *_1)_{\tau\omega}$

- does not restrict the set of formulas the agent ‘knows’, but ***computes*** the ***inferable*** knowledge relative to the ***rule(s)*** the agent is *able* to use
- Exacting, demanding approach

Computing inferable knowledge:

- Inf^a / $((o\ *_n)\ (o\ *_n))_{\tau\omega}$ – an agent's *inferential abilities*
  - *b* — *a* set of constructions: knowledge of *a*
  - ***c*** — inferred construction: inferable piece of knowledge
  - $b \rightarrow (o\ *_n)$, $c \rightarrow *_n$, $d \rightarrow *_n$
  - $|\!\!-\!\!-_r$ derivation according to the rule *r that a* masters (is able to use)
- $Inf^a : \lambda w\ \lambda t\ \lambda b\ \lambda c\ [[b\ c] \vee [\exists r\ (b\ |\!\!-\!\!-_r\ c)]$

Computing inferable knowledge:

- ***Recursive definition***
  (omitting trivialisations if 'used'):
  - $\mathbf{K_0{}^a{}_{wt} = Kexp^a{}_{wt}}$
  - $\mathbf{K_{n+1}{}^a{}_{wt} = [\ Inf^a{}_{wt}\ K_n{}^a{}_{wt}\ ]}$
  - *Nothing other…* $\mathbf{Kinf^a{}_{wt} = \cup_j K_j{}^a{}_{wt}}$
- $Kinf^a{}_{wt} = [\ Inf^a{}_{wt}\ Kinf^a{}_{wt}\ ]$ – fixed point of Inf^a
- *Monotonic reasoning* – **the least fixed point**:
  - $\mathbf{Kinf^a{}_{wt} = \mu\lambda x\ [\ Inf^a{}_{wt}\ [\ x \cup Kexp^a{}_{wt}\ ]\ ]}$

Computational semantics in details

- There are technical problems here: we need to mention the construction by trivialising it:
  - ‘calling a sub-procedure with formal parameters *c, d*’.
- To release variables *c, d* bound by trivialisation, we have to use special substitution functions SUB:
  - substituting actual values for formal parameters.
- The upper index *c,d* is a notational abbreviation of these facilities; (‘talking about’ the objects of attitudes)
- double-executing variables ranging over constructions of propositions, ^2c, ^2d, returns the respective propositions.
- $(^2d)_{wt}$ — intensional descent: constructs a truth-value

Web ontology languages

- Lack of precise semantics
- Do not communicate with each other and understand each other
- The terminology is not unique, sometimes even incompatible
- *Built bottom up*: FOL approach enriched by constructs as needed
- Attempts at standardization: W3C group

Three groups of ontology languages

- RDF (*Resource Description Framework*) and RDFS(chema) languages
- OIL, DAMT-ONT a DAML+OIL $\rightarrow$
  OWL (*Web Ontology Language*)
- SKIF a *Common Logic*

Generally, they do not enable us to express:

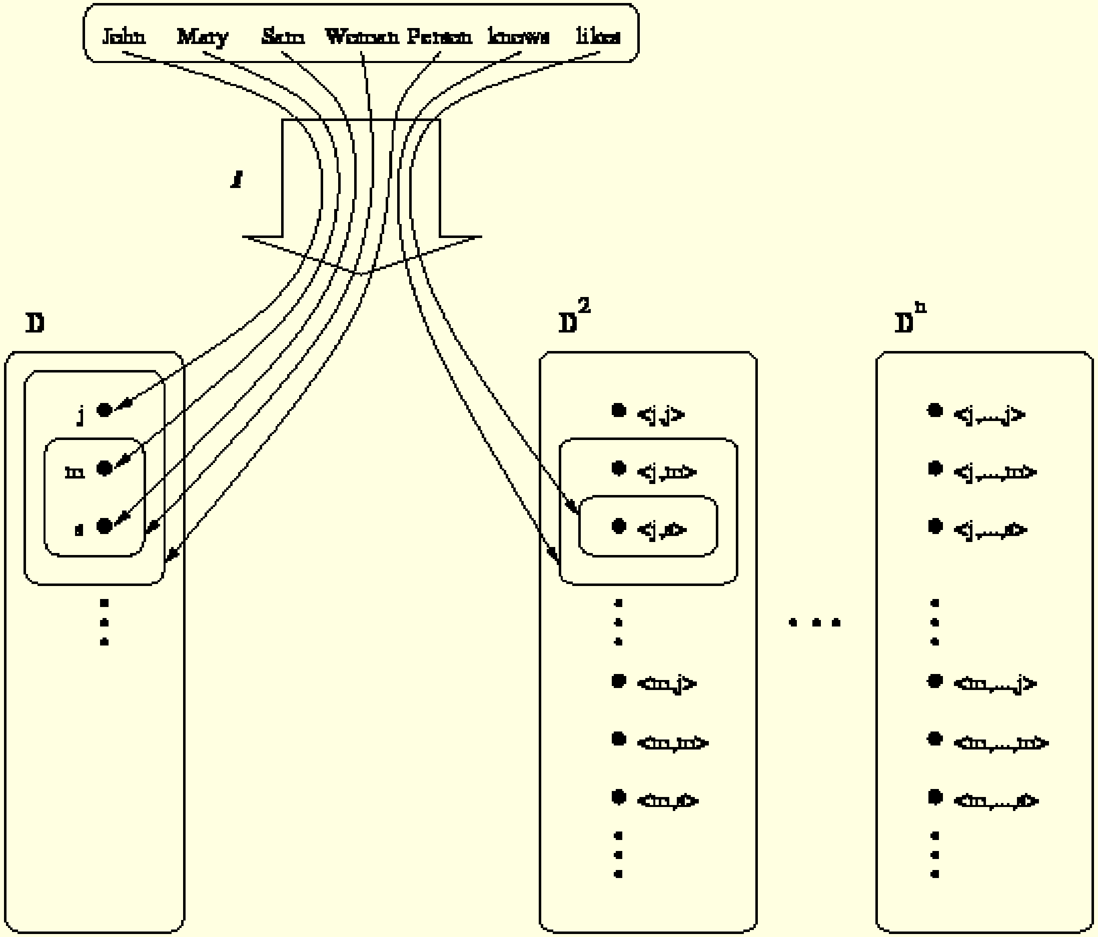
- Modalities: what is ***necessary*** and what is ***contingent***
- ***Analytic*** vs. ***empirical concepts***
- ***n-ary relations*** (only binary relations – unreasonably called *properties*)
- To talk about concepts

RDFS: *W3C standard* !?

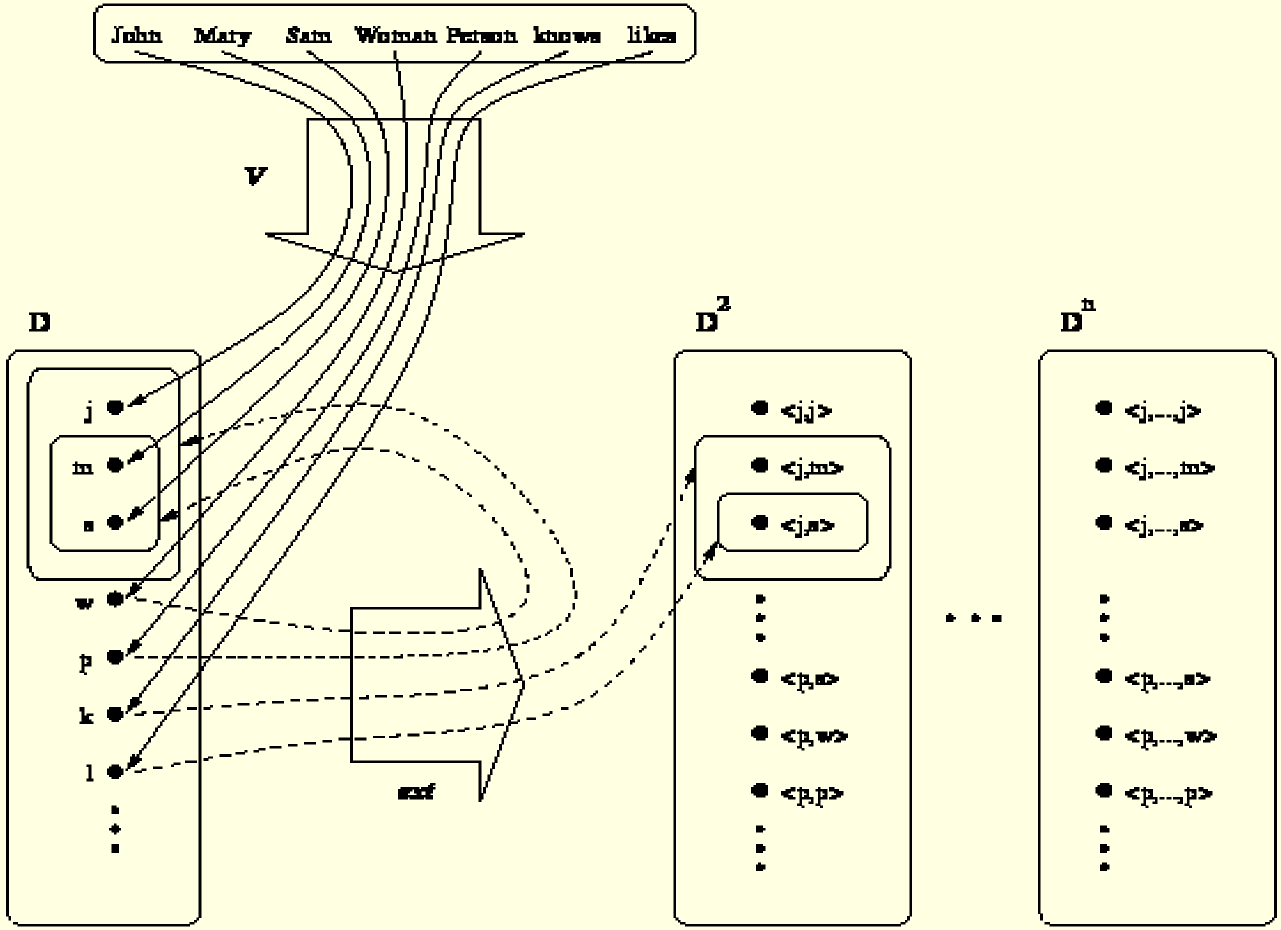
- Low expressive power
- Originally no formal semantics
- Language of “triples”: subject – predicate -- object

- OWL – syntactically compatible, but ***not*** semantically (based on Description logic)
- SKIF – enables us to express some ***higher-order*** propositions, e.g.:
  - John and Mary have some common property – $\exists x.x(\text{Mary}) \,\&\, x(\text{John})$
  - P is valid of *x* and Q is valid of P

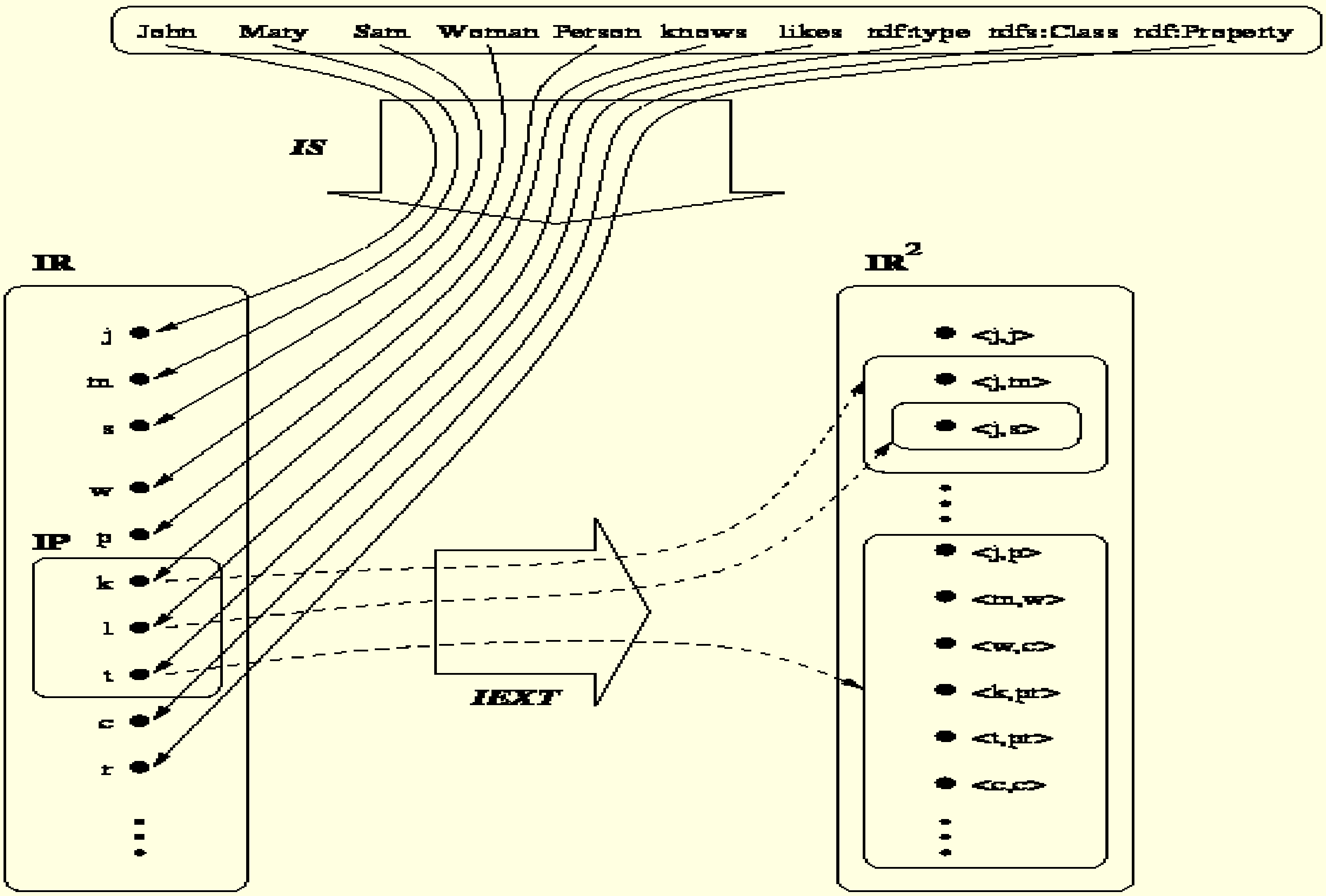
FOL semantics



SKIF semantics



RDFS Thesis


John Mary Sam Woman Person knows likes rdf:type rdfs:Class rdf:Property
IS
IR
IR²
j
m
s
w
p
IP
k
l
t
c
r
IEXT
<j,j>
<j,m>
<j,s>
<j,p>
<m,w>
<w,c>
<k,pr>
<t,pr>
<c,c>

Description logic

- FOL approach, rather rich, makes it possible to distinguish:
  - ‘extensional’ knowledge (of ***contingent*** facts) A-boxes vs.
  - ‘intensional’ (general, ***necessary***, relations and properties of concepts) – T-box ***definitions***:
    - Satisfiable concepts (non-empty)
    - Subsumption: whale vs. mammal,
    - Equivalence, and
    - Disjointness of concepts: bachelor vs. married man

Theory of Concepts

- TIL: Concept is a closed construction.
- *Examples*:
  a) $\lambda w\ \lambda t\ \lambda x\ [[{}^0\text{Greatest}\ [{}^0\text{Sea}\ {}^0\text{Mammal}]]_{wt}\ x]$
  b) ${}^0\text{Whale}$
     - Equivalent concepts: concept a) is a *definition* of the property Whale
  - *All whales are mammals*
     - analytically true proposition: *Necessarily, if anything is a whale, then it is a mammal:*
     - $[{}^0\text{Requisite}\ {}^0\text{Mammal}\ {}^0\text{Whale}] = \forall w\ \forall t\ \forall x\ [[{}^0\text{Whale}_{wt}\ x] \supset [{}^0\text{Mammal}_{wt}\ x]]$

Theory of Concepts

- *Subsumption*: ^{0}whale subsumes ^{0}mammal, for
  [^{0}Requisite ^{0}Mammal ^{0}Whale]

- *Equivalence*
  the two concepts identify the same entity
  ^{0}bachelor =
  $\lambda w \lambda t\ \lambda x\ [[[^0elder\ ^0man]_{wt}\ x] \wedge \neg[^0married_{wt}\ x]]$

- *Disjointness* of concepts:
  bachelor vs. married man

- TIL: not only constructions of properties (‘universals’), but also individual concepts, concepts of a proposition, …

Theory of Concepts

- TIL: Kinds of *emptiness*.
  - Strictly empty: the greatest prime
  - Empty: round square, married bachelor
  - (Empirically) empty at *w*,*t*: president of CR (in January 2003)
- *Satisfiable concepts:* non-empty (in some *w*, *t*) – all 'non-contradictory' concepts
  - $\lambda w \lambda t\ [{}^0President_{wt}\ {}^0CR]$, $\lambda w \lambda t\ [{}^0King_{wt}\ {}^0France]$
  - $\lambda x\ [[{}^0prime\ x] \wedge [{}^0divisible\ x\ {}^02]]$

Should be recommended for Semantic web

- Higher-order logic (HOL) approach: comprehensive, natural way of expressing knowledge
- Objections: “too complicated”?
  - Complexity concerns problems not language
- Semantic incompleteness: do we need a complete axiomatization?
  - We need a high expressive power and
  - natural deduction from assumptions to compute inferable knowledge